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APERTURE CURRENTS AND PROBE RESPONSES FROM ELECTROMAGNETIC TESTS

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by

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NOMENCLATURE

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English Letters

A	Area
e	Base of the natural logarithms
f	Complex potential function
H	Magnetic field intensity
i	Electric current
i_i	Current enclosed by the inner streamline of a tube of flow
i_o	Current enclosed by the outer streamline of a tube of flow
i_{or}	Current within a tube of flow
J	Current density
J_o	Current density with no aperture
J_t	Total current density
j	The complex operator $\sqrt{-1}$
K_1, K_2, k	Arbitrary constants
Re[ ]	Real part of [ ]
r	Radius in polar coordinates
r_1	Pickup coil radius
U, V	Some physical quantities
v	Charge velocity
x	Real part of z
y	Imaginary part of z
z	Complex variable in distance units

Greek Letters

α	Centerline direction from aperture to pickup coil relative to tube axis
β	Angular half-spread of aperture and pickup coil overlap
θ	Angle in polar coordinates
μ	Permeability
ρ	Volume charge density
φ	Scalar potential function
ψ	Stream function

Subscripts

r	Radial component
θ	Tangential component
x	x-axis component
y	y-axis component
z	z-axis component

Special Marks

$\rightarrow$	Vector quantity
$\hat{}$	Unit vector
∇	Vector del operator
$*$	Complex conjugate

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
H	Magnetic field intensity
I	Electron current
I_i	Current enclosed by the inner streamline of a tube of flow
I_o	Current enclosed by the outer streamline of a tube of flow
I_{tf}	Current within a tube of flow
J	Current density
J_0	Current density with no aperture
J_t	Total current density
j	The complex operator $\sqrt{-1}$
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ABSTRACT

A mathematical validation is presented for representing apertures and defects in conductors as current sources when excited by massive externally pulsed fields. The current distribution contributed by a cylindrical void in a conductor is derived, and the result is used to approximate the flux that links a pickup coil, the rim of which passes over an aperture. The concept of defect current is used to estimate the form of the response of a scanning mask-aperture probe passing over a defect.

INTRODUCTION

Predicting the behavior of mask-aperture-type probes, for pulsed electromagnetic systems, at the design stage continues to depend primarily on empirical "try-and-see" methods that use the intuition of experienced designers. Precise mathematical descriptions of the processes involved are as yet incomplete, though promising work[1] is in progress.

The complicated boundary conditions and the complex field geometries inherent to these probes present formidable problems for the analyst. It is difficult to achieve solutions that are sufficiently general to guide the designer toward improved designs. Analyses that attempt to verify the observed behavior of an empirically designed probe continue to be based on hindsight. Thus each analysis is for a special case and of little help in improving designs.

This report attempts to broaden the designer's visualization of the process. A previous report[2] discussed the diffusion of currents in mask and sample from a lumped-parameter approach based on plane-wave analysis, and provided the designer with a qualitative circuit-like description of the process. Lately, it has become customary and useful for the designer to regard the perturbations due to defects as arising from "defect currents" induced into field geometries that were otherwise relatively uniform in the wall. An objective of this report is to provide an analytical basis for this useful concept, and its effect on the signals from scanning probes is discussed. The results provide a qualitative glimpse of a complex process.

CURRENT FLOW AROUND AN APERTURE

In a metal, which is charge neutral, the continuity equation for current is

$$\nabla \cdot \vec{J} = \nabla \cdot (\rho\vec{v}) = 0, \tag{1}$$

where J and v are the current-density and charge-velocity vectors, and ρ is the volume charge density. The current density can be considered the gradient of some scalar potential, i.e.,

$$\vec{J} = \nabla\varphi, \tag{2}$$

since φ satisfies the Laplacian

$$\nabla^2\varphi = 0. \tag{3}$$

Assume that φ is independent of one of the three space coordinates so that it can be expressed as

$$\varphi = \varphi(x,y). \tag{4}$$

Using complex variable theory, such cases can be handled conveniently by defining a <u>complex current</u> (or <u>charge velocity</u>) <u>potential</u> that has φ as its real part. When the complex variable is

$$z = x + jy, \tag{5}$$

the complex potential is

$$f(z) = \varphi(x,y) + j\psi(x,y). \tag{6}$$

If the above is analytic, the following Cauchy-Riemann conditions hold:

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y}, \tag{7}$$

and

$$\frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}. \tag{8}$$

From these equations it is clear that

$$\frac{\partial^2\varphi}{\partial x^2} = -\frac{\partial^2\psi}{\partial y^2},$$

and

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \nabla \cdot \nabla\varphi = \mathrm{Re}[\nabla(\nabla\varphi)^*] = \nabla^2\varphi = 0. \tag{9}$$

Here we have used the fact that when two-dimensional vectors are represented by complex numbers, the inner (or dot) product of the two vectors

$$\vec{U} = U_x + jU_y \text{ and } \vec{V} = V_x + jV_y$$

is

$$\vec{U} \cdot \vec{V} = \mathrm{Re}[UV^*] = U_xV_x + U_yV_y,$$

where the asterisk indicates the conjugate.

The Cauchy-Riemann conditions show that

$$\nabla\varphi \cdot \nabla\psi = \vec{J} \cdot \nabla\psi = 0. \tag{10}$$

This means that the lines of constant ψ and constant φ are mutually orthogonal; thus the constant ψ lines are <u>streamlines</u> for the current flow.

The flow of current around a nonconducting void or intrusion can be considered laminar, or nonturbulent, flow. The "fluid" in the flow is incompressible, because charge density in a conductor does not change with time. Such flow around a cylindrical obstruction can be described by a potential function that is well known[3] and that provides the jumpoff material for the following development.

Assume z to be a point in the complex plane representing distance normalized with respect to some convenient base distance. Then

$$z = x + jy = re^{j\theta}, \tag{11}$$

and we can define the complex current potential as

$$\begin{aligned} f(z) &= J_o\left(z + \frac{1}{z}\right) \\ &= J_o\left(x + jy + \frac{1}{x + jy}\right) \\ &= J_o\left(x + \frac{x}{x^2 + y^2}\right) + jJ_o\left(y - \frac{y}{x^2 + y^2}\right) \\ &= (x,y) + j(x,y). \end{aligned} \tag{12}$$

The Cauchy-Riemann conditions are satisfied because

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial z} = J_o\left[1 - \frac{x^2 - y^2}{(x^2 + y^2)^2}\right],$$

$$\frac{\partial\varphi}{\partial y} = -\frac{\partial\varphi}{\partial x} = \frac{-2J_o xy}{(x^2 + y^2)^2}, \tag{13}$$

and $\varphi(x,y)$ is a valid current potential. The current density is

$$\vec{J}_t = \nabla\varphi = \frac{\partial\varphi}{\partial x} + j\frac{\partial\varphi}{\partial y} = J_{tx} + jJ_{ty}$$

$$= J_o\left[1 - \frac{x^2 - y^2}{(x^2 + y^2)^2}\right] - j\frac{2J_o xy}{(x^2 + y^2)^2}. \tag{14}$$

For distances far from the origin, where z is large, the current density thus defined approaches J_o. That is,

$$\vec{J}_t \rightarrow J_o + j0 \text{ as } r = |x| = |x + jy| \rightarrow 0. \tag{15}$$

Any streamline (ψ = constant) can be considered a boundary between two regions, one a nonconducting region obstructing the current flow and the other filled with streamlines representing flow around the obstruction. Consider the unit circle, where

$$z = e^{j\theta} = \cos\theta + j\sin\theta.$$

On this circle

$$f(z) = J_o\left(\cos\theta + j\sin\theta + \frac{1}{\cos\theta + j\sin\theta}\right)$$

$$= J_o(\cos\theta + j\sin\theta + \cos\theta - j\sin\theta)$$

$$= 2J_o\cos\theta + j0.$$

Thus the unit circle represents the streamline $\psi = 0$, and the inside of the circle can be considered a nonconducting region. The streamlines outside the unit circle can be considered as the flow around a cylindrical nonconducting region within a conductor that would otherwise be supporting the uniform conduction. Equation 15 indicates this conduction would be of density J_o in the positive x direction, which is illustrated in Fig. 1.

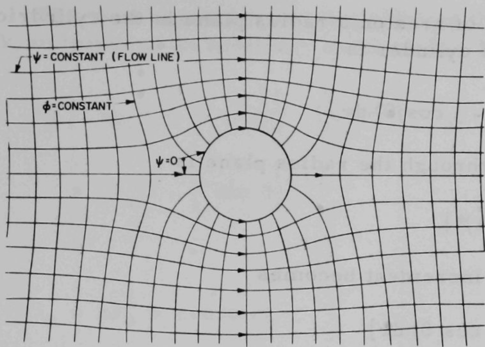

Fig. 1
Current Flow around a Circular Aperture. Neg. No. MSD-55159.

APERTURE CURRENTS DISTURBING UNIFORM FLOW

The current distribution around the aperture in Fig. 1 may be considered the superposition of two separate distributions, one the original uniform flow $J_o + j0$, the other a hypothetical flow J due to the aperture. Thus the flow in Fig. 1 and in Eq. 14 is

$$\vec{J}_t = J_o + \vec{J},$$

and the flow outside the aperture due to the aperture is, by Eq. 14,

$$\begin{aligned}
\vec{J} &= \vec{J}_t - J_o \\
&= \frac{-J_o(x^2 - y^2)}{(x^2 + y^2)^2} - j\,\frac{2J_o xy}{(x^2 + y^2)^2} \\
&= -J_o\,\frac{x^2 - y^2 + j2xy}{(x^2 - y^2)^2} = -J_o\,\frac{(x + jy)^2}{(x^2 + y^2)^2} \\
&= -\frac{J_o}{r^2}e^{2j\theta}, \qquad r > 1.
\end{aligned} \tag{16}$$

The flow inside the aperture due to the aperture must just cancel the original flow. Therefore, the complete expression for "aperture-produced" current is

$$\vec{J} = \begin{cases} -J_o + j0, & 0 < r < 1 \\ -\dfrac{J_o}{r^2}e^{j\theta}, & 1 < r < \infty. \end{cases} \tag{17}$$

The vector increment of area on a radius plane in the cylindrical coordinates per unit length of cylinder is

$$d\vec{A} = -jre^{j\theta} = (\sin\theta - j\cos\theta)\,dr, \tag{18}$$

and the incremental current through the radius plane is

$$di = \vec{J}\cdot d\vec{A} = \mathrm{Re}[\vec{J}(d\vec{A})^*]. \tag{19}$$

Inside the unit cylinder, this increment becomes

$$di = \mathrm{Re}[-J_o(\sin\theta + j\cos\theta)\,dr]$$

$$= (-J_o \sin\theta)\,dr.$$

Outside the unit cylinder,

$$di = \mathrm{Re}\left[\frac{J_o}{r^2}\,e^{j2\theta}(+je^{-j\theta})\,dr\right]$$

$$= \mathrm{Re}\left[-j\frac{J_o}{r^2}\,e^{j\theta}\,dr\right]$$

$$= \frac{J_o}{r^2}\sin\theta\,dr.$$

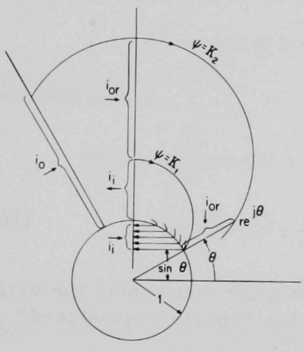

Fig. 2. Division of Aperture Currents. Neg. No. MSD-55154.

Summarizing,

$$di = \begin{cases} -J_o \sin\theta\,dr, & 0 < r < 1 \\ \dfrac{J_o}{r^2}\sin\theta\,dr, & 1 < r < \infty. \end{cases} \tag{20}$$

Refer now to Fig. 2 from Eq. 20,

$$i_i = \int_{\sin\theta}^{1}\left(-J_o \sin\frac{\pi}{2}\right)dr = -J_o(1 - \sin\theta), \tag{21}$$

and

$$i_{or} = \int_1^r \frac{J_o}{\rho^2}\sin\theta\,d\rho = -J_o\left(\frac{1}{r} - 1\right)\sin\theta. \tag{22}$$

The total current i_o flowing between the point P and the streamline $\psi = k$, in terms of an arbitrary point on $\psi = k$, is

$$i_o = i_{or} - i_i = -J_o\left(\frac{1}{r} - 1\right) \sin\theta + J_o(1 - \sin\theta)$$

$$= -J_o\left(i - \frac{1}{r}\sin\theta\right). \tag{23}$$

Let

$$i_o = kJ_o = \text{constant}.$$

Then

$$k = 1 - \frac{1}{r}\sin\theta, \tag{24}$$

and

$$r = \frac{\sin\theta}{1 - k}.$$

The flow line on which $i_o = kJ_o$ is defined by

$$z = \frac{\sin\theta}{1 - k} e^{j\theta}$$

$$= \frac{1}{j2(1 - k)}\left(e^{j\theta} - e^{-j\theta}\right) e^{j\theta}$$

$$= \frac{j}{2(1 - k)}\left(1 - e^{j2\theta}\right). \tag{25}$$

This is the equation of a circle with

$$\text{center at } z = \frac{j}{2(1 - k)},$$

and

$$\text{radius} = \frac{1}{2(1 - k)}. \tag{26}$$

The circles of constant ψ will all be tangent to the x axis if extended within the unit circle (where they are not valid). It should be noted that the normalizing distance is the radius of the cylinder in lineal units. If this radius shrinks to zero (a point aperture), then the circles do approach the point of tangency at the origin, and we have a true current dipole.

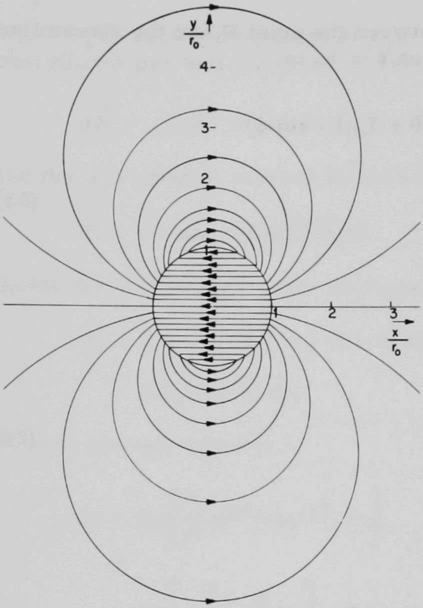

Fig. 3. Aperture-current Distribution.
Neg. No. MSD-55155.

The defect current distributions, plotted for

$$k = 0.1n, \qquad n = 1, 2, \ldots, 10,$$

are shown in Fig. 3, where x and y are real-dimensioned quantities, and r_o is the radius of the aperture. The amount of current flowing between each pair of flow lines (cylinders) is equal to one-tenth the total current through the aperture.

Equation 17 may be changed to conventional cylindrical coordinates by noting that the x-y plane unit vectors in that system may be expressed as

$$\hat{r} = \frac{\vec{r}}{r} = e^{j\theta}, \quad \hat{\theta} = je^{j\theta}.$$

Thus

$$1 = (\cos\theta)e^{j\theta} - j(\sin\theta)e^{+j\theta}$$

$$= (\cos\theta)\hat{r} - (\sin\theta)\hat{\theta},$$

and

$$e^{j2\theta} = (\cos\theta + j\sin\theta)e^{j\theta} = (\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}.$$

Equation 17 in cylindrical coordinates is then

$$\vec{J} = \begin{cases} -(J_o\cos\theta)\hat{r} + (J_o\sin\theta)\theta, & 0 < |r| < 1 \\ -\left(\frac{J_o}{r^2}\cos\theta\right)\hat{r} - \left(\frac{J_o}{r^2}\sin\theta\right)\hat{\theta}, & 1 < |r| < \infty \end{cases}$$

$$= J_r(r,\theta)\hat{r} + J_\theta(r,\theta)\hat{\theta}. \tag{27}$$

Along the infinite plane, θ = constant = $\pm\pi$,

$$H_z(r,\theta) = -\int_{-\infty}^{r} J_o(r,\theta)\,dr = \begin{cases} J_o r\sin\theta, & |r| < 1 \\ -\frac{J_o}{r}\sin\theta, & 1 < |r| < \infty, \end{cases} \tag{28}$$

which is plotted in Fig. 4.

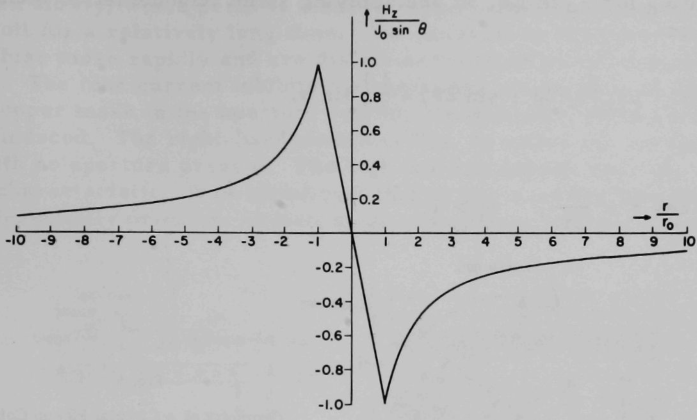

Fig. 4. Variation in Field Strength across a Cylindrical Aperture. Neg. No. MSD-55157.

PICKUP-COIL RESPONSE TO APERTURE CURRENT

With J_o to the right, as shown in Fig. 5, Eq. 28 becomes

$$H_z = \begin{cases} J_o r \sin\theta, & r < 1 \\ \dfrac{J_o}{r} \sin\theta, & r < 1. \end{cases} \tag{29}$$

Thus the flux that links the pickup coil is

$$\varphi = \int_A H_z \, dA.$$

If the small area between the lines at $\theta = \alpha \pm B$ and that part of the pickup coil over the aperture are neglected, this becomes

$$\varphi = \mu J_o \left[\int_{\alpha-\beta}^{\alpha+\beta} \int (r \sin\theta) \, dr \, dr \, \theta + \int_{\alpha-\beta}^{\alpha+\beta} \int_1^{r_\theta} \left(\frac{1}{r} \sin\theta \right) dr \, dr \, \theta \right]$$

$$= \mu J_o \left[-\frac{2}{3} \int_{\alpha-\beta}^{\alpha+\beta} \sin\theta \, dr \, \theta + \int_{\alpha-\beta}^{\alpha+\beta} r_\theta \sin\theta \, dr \, \theta \right]. \tag{30}$$

From the geometry in Fig. 5,

$$r_\theta = 2 r_1 \cos(\alpha - \theta). \tag{31}$$

Substituting for r_θ in Eq. 30 and applying some trigonometric identities results in

$$\varphi = \mu J_o \left[r_1(2\beta + \sin 2\beta) - \frac{1}{3} \right] \sin \alpha, \tag{32}$$

and

$$\cos \beta = \frac{1}{2r_1}, \qquad r_1 > 1. \tag{33}$$

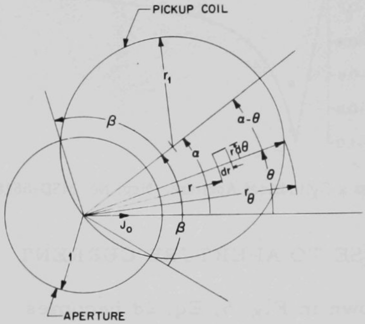

Fig. 5

Geometry of a Circular Pickup Coil over a Cylindrical Aperture. Neg. No. MSD-55153.

Since r_1 and, therefore, β are design parameters for a given configuration of pickup and aperture, the flux that links the pickup coil is proportional to the density of the mask current and the sine of its direction relative to the line connecting the centers of the aperture and coil. That is,

$$\varphi = kJ_o \sin \alpha. \tag{34}$$

THE MULTIAPERTURE PROBE

As can be seen from Fig. 6, the probe and the tube under test present an electromagnetic problem with troublesome boundary conditions. The encircling exciting coil is driven by a massive current pulse that produces an intense and rapidly changing magnetic field. Initially, surface currents are induced in those portions of the mask and tube nearest the exciting coil, and these currents produce fields in opposition to those of the exciting coil. The induced surface currents diffuse, with the passage of time, along the metal away from the exciting coil. The rate of diffusion depends on the conductivity of the metal, the presence of defects, and nearby currents in other conducting members.[2]

Figure 6a, on the left, shows a possible distribution of currents in mask and tube that results from an exciting current pulse when no defect is present. The high conductivity of the copper mask causes the currents to diffuse slowly, which permits them to remain concentrated near the exciting coil for a relatively long time. The currents in the lower-conductivity tube diffuse more rapidly and are distributed further away from the exciting current. The tube current inhibits magnetic-flux penetration of the surface of the copper mask in the aperture region; consequently, little surface current is induced. The right-hand sketch in Fig. 6a shows the current flow lines with no aperture present. The high current density near the exciting coil is characteristic. It is significant that in this case the flow lines are circumferentially oriented, without axial components to their direction.

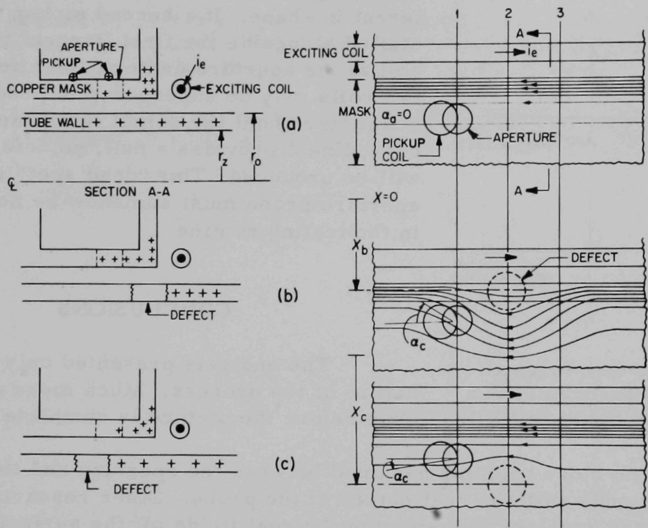

Fig. 6. Approximate Effect of a Nonconducting Cylindrical Void on Mask-current Distributions in a Mask-aperture Probe. Neg. No. MSD-55158.

Figure 6b shows the effect of a defect near the exciting coil. The absence of tube current at the defect allows the penetration of an additional portion of the mask by magnetic flux with a resultant inducing of currents in the mask further away from the exciting current. Inasmuch as the total induced current remains essentially unchanged, the flow lines are diverging in the neighborhood of the defect, as shown in the right-hand part of the figure. Definite axial components to the flow-line directions appear to the right and left of the defect.

Figure 6c represents an estimate of the effect of the defect moving out of the conducting regions. The disturbance should now be small, except possibly for some slight convergence of flow lines as shown.

In the figure, the axial lines 1, 2, and 3 on the mask represent possible aperture locations relative to the flow. Along line 1, the moving defect causes the mask current to flow at a changing angle α with respect to the line between aperture and pickup-coil centers, and some change in aperture-location flux density is caused. Application of Eq. 34 to the current fields in the figure, using the approximation of uniform no-aperture current distribution in the directions indicated by α_a, α_b, and α_c, will yield aperture flux variations similar to those of Fig. 7.

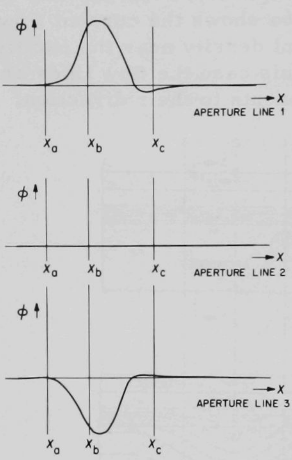

Fig. 7. Approximate Pickup-coil Flux-linkage Variations in Fig. 6. Neg. No. MSD-55156.

Distributions along aperture lines 1 and 2 are negative to one another and somewhat different in shape. If a second pickup coil is installed alongside the first (tangent to the first coil at the aperture center), the effects of the two coils may be superposed to produce antisymmetry about the defect line (line 2). In any case, line 2 provides a null; no defect indication will be produced. This "dead spot" in the mask-aperture probe must somehow be accounted for in the testing routine.

CONCLUSIONS

The analysis presented only suggests the nature of the process. Much more needs to be done before the picture is complete.

The close coupling assumed between the aperture and the pickup coil is at odds with the real nature of the probe. Other researchers are at work on the problem of evaluating the real fields off the surface of the mask,[4] and the successful culmination of such efforts will yield a quantitative replacement for the qualitative results discussed in the present report.

The obvious fact that defects are not dependably cylindrical, as assumed here, should be pointed out. However, the possibility exists that such primitive configurations, perhaps infinitesimal in size, may be used to build up the effects of more complicated defects, such as cracks of finite lengths. There have been field calculations based on magnetic dipoles, the equal and opposite magnetic poles of which have been assumed distributed on the opposing walls of a defect, i.e., a crack.[5] A similar analysis based on current dipoles, formed as suggested within this report, might provide useful results.

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